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M-theory Compactifications on Manifolds with G_2 Structure

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Abstract

In this paper we study M-theory compactifications on manifolds of G_2 structure. By computing the gravitino mass term in four dimensions we derive the general form for the superpotential which appears in such compactifications and show that beside the normal flux term there is a term which appears only for non-minimal G_2 structure. We further apply these results to compactifications on manifolds with weak G_2 holonomy and make a couple of statements regarding the deformation space of such manifolds. Finally we show that the superpotential derived from fermionic terms leads to the potential that can be derived from the explicit compactification, thus strengthening the conjectures we make about the space of deformations of manifolds with weak G_2 holonomy.

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1 Introduction

Low-energy M-theory solutions with seven compact and four large dimensions that yield $N = 1$ supergravities play an important role in the 'landscape' of string and M-theory vacua. As is always the case, lower dimensional supersymmetry imposes strong constraints on the internal manifold. In particular this should posses at least one globally defined spinor or—equivalently—have its structure group contained in $G_2 \subset SO(7)$ [1]. If one also requires that the four-dimensional space-time be flat then the internal manifold must be Ricci flat and have holonomy group contained in G_2 [2].

If, on the other hand, one allows for a curved four-dimensional space-time whilst still preserving maximal symmetry and supersymmetry in four dimensions, then the external space becomes AdS. The internal manifold then need not be Ricci flat and will typically be an Einstein space of constant positive curvature. Solutions of this type were extensively considered after the discovery of $D = 11$ supergravity and are known as Freund–Rubin spontaneous compactifications [3]. Much of this earlier work is reviewed in [1]. Using the elegant notions of G-structures, the internal spaces for these solutions that support a single globally defined spinor will be of the so-called weak G_2 type [4, 5].

The compactifications described above suffer from two major problems, which are common to most string/M-theory compactifications. First of all, they give rise to many moduli in four dimensions. These are massless scalar fields which are exact flat directions of the potential and parameterise a huge vacuum degeneracy. Second, the spectrum of fermions in four dimensions is in general non-chiral – and thus not suitable for particle physics phenomenolygy – due to a quite general index theorem for smooth seven-dimensional spaces [6].

As became customary in the last period, the first problem can be tackled by turning on non-trivial background fluxes which induce a (super)potential for the scalar fields [7, 8]. There was also recent work on moduli fixing for M-theory on G_2 manifolds including non-perturbative contributions in [9].

The resolution of the second problem came with the advent of dualities and the realisation that chiral fermions appear at singular conical points in the seven-dimensional manifold [10, 11], together with a mechanism for the cancellation of the appropriate anomalies [12].[1] Conical singularities are quite well understood in the non-compact case [5], but so far it has proved difficult to construct compact G_2 holonomy manifolds with point-like singularities. This is partly because the standard way in which such manifolds are constructed is by the orbifolding of seven-tori followed by the blowing up of the orbifold singularities [15, 16, 17], which does not naturally produce point-like singularities.

The known examples of compact manifolds with codimension seven singularities are not in fact manifolds with G_2 holonomy, but weak G_2 manifolds, and it is known [5] that chiral fermions can also emerge from singular weak G_2 manifolds. These manifolds can be constructed by adding an extra compact dimension to weak $SU(3)$ spaces to form a 'lemon' with two conical singularities [18]. Although having two singularities presents phenomenological problems, spaces such as these may play a role as simple models, and motivate further study into general features of weak G_2 manifolds.

The presence of background fluxes—when not perturbatively small—in M-theory as well as in string compactifications induces a back-reaction on the underlying geometry so that manifolds of

[1] Such compactifications turn out to be dual to intersecting brane models in the context of type IIA string theory, as originally discussed in [13, 14].

G_2 holonomy are no longer solutions to the equations of motion. In turn the resulting geometry can be described in terms of the more general concept of G-structures [4][19]–[40]. Compactifications on manifolds with non-trivial G structure are poorly understood, however, mainly because of the lack of knowledge of the deformation space of these manifolds and it will be our purpose in this paper to clarify some aspects of these compactifications for the particular case of manifolds with G_2 structure. In several works it was proposed that a superpotential which depends on the structure is generated, but in most cases the expression of these superpotentials in terms of the low energy fields and their relevance for moduli stabilisation was not possible to compute due to the lack of understanding of the low energy degrees of freedom which appear in such compactification.

A different route to understand G-structures in string and M-theory compactifications is via dualities [41]–[51]. In this way it is possible to deduce certain properties of the moduli space of these manifolds. To our knowledge this was only done in [41] where the low energy action was also computed. For such models an explicit expression for the superpotential in terms of the low energy fields can be found [49] and thus it is possible to say something about the dynamics (and in particular moduli stabilisation). The main assumption in these papers was that the fluxes/intrinsic torsion are perturbatively small and that in such a regime the moduli space is similar to the moduli space of some Calabi–Yau manifold. The generalisation of this conjecture for large fluxes/intrinsic torsion is, however, not known.

In a recent work, another conjecture about the deformation space of nearly-Kähler manifolds (known also as manifolds of weak $SU(3)$ holonomy) was made [51]. In this paper it was proposed that the weak $SU(3)$ conditions impose strong constraints on the space of possible perturbations of such manifolds and only size deformations of these manifolds are allowed.

The purpose of this paper is twofold. In the first instance we study some general aspects of M-theory compactifications on manifolds with G_2 structure. By compactifying certain fermionic terms we derive the general form of the superpotential which appears in such compactifications. The formula we obtain generalises in a natural way the result obtained in [7] for just F_4 fluxes based on the general analysis in [52, 53]. Motivated by the fact that the manifolds with weak $SU(3)$ holonomy are tightly constrained, in the second part of this paper we study their seven-dimensional relatives, namely manifolds with weak G_2 holonomy. Such manifolds have the property that the intrinsic (con)torsion is a singlet under the structure group G_2 and it turns out that one can infer enough about their internal structure to allow us to derive the low-energy effective action for M-theory compactified on such manifolds. In the end we link the two parts of the paper by showing that the general formula for the superpotential derived in general for any G_2 structure produces the correct result for the manifolds with weak G_2 holonomy in that the potential which is obtained from the compactification can be also obtained from this superpotential when inserted in the standard $N = 1$ formula.

M-theory on manifolds with restricted structure group was also studied in [4, 54]. Here it was shown among other things that if the structure group is exactly G_2 the only supersymmetric solution is given by the Freund–Rubin compactification [1] and that the internal manifold must be of weak G_2 type. In this paper we go one step further and present the generic form of the low-energy theory obtained in such compactifications. It is important to observe that the four-dimensional ground state in this case is AdS which together with the presence of a non-trivial flux along the four space-time dimensions changes the definition of the mass operators for the fields which appear in the low-energy theory. This should tell us that the appropriate AdS massless modes no longer

appear when the fields are expanded in harmonic forms, but in forms which satisfy

$$d\alpha_{(3)} = -\tau * \alpha_{(3)} \ , \tag{1.1}$$

where τ measures the intrinsic torsion of the manifold with weak G_2 holonomy. It will also turn out that these are precisely the variations of the G_2 structure which are compatible with the weak G_2 conditions. Thus one can see in the same way as on manifolds with G_2 holonomy that the modes coming from the matter fields combine with the modes coming from the metric into the complex scalars $a_3 + i\varphi$.

The organisation of the paper is the following. In section 2 we present our conventions for the M-theory action and G_2 structures. In section 3 we derive the gravitino mass term which appears in compactifications on manifolds with G_2 structure which will give us the Kähler potential and the superpotential for the chiral fields coupled to supergravity in four dimensions. In section 4 we discuss weak G_2 manifolds and concentrate on their deformation space. Using this information we perform in section 5 the Kaluza–Klein reduction on manifolds with weak G_2 holonomy and show agreement with the general results derived in section 3 for the superpotential and Kähler potential. In section 6 we present our conclusions. For completeness we have also assembled a couple of appendices where we present our conventions and the most useful relations on manifolds with G_2 structure which we use throughout the paper. Here we have also included some of the technical details which were skipped in the main part of the paper.

2 Preliminaries

2.1 Action and Ansatz

We start by introducing our M-theory conventions which we will use throughout the paper. As it is well-known, the low-energy approximation of M-theory is given by the eleven-dimensional supergravity which describes the dynamics of the $N = 1$ supermultiplet in eleven dimensions. This contains the metric $\hat{g}_{MN}$ and an antisymmetric tensor field $\hat{A}_{MNP}$ as bosonic components and the gravitino $\hat{\Psi}_M$, which is a Majorana spinor in eleven dimensions, as their fermionic superpartner. Hats are used to distinguish the eleven-dimensional fields from the four- and seven-dimensional ones that we will introduce later on. We use D for the spinor covariant derivative. The indices $M, N \ldots = 0, 1, \ldots, 10$ denote curved eleven-dimensional indices. The supergravity action can then be written as in [55]

$$\begin{aligned} S \;=\;& \frac{1}{2}\int_{M_{11}} \sqrt{-\hat{g}}\; d^{11}x \left[\hat{R} - \overline{\hat{\Psi}}_M \hat{\Gamma}^{MNP} D_N \hat{\Psi}_P - \frac{1}{2}\hat{F}_4 \wedge *\hat{F}^4\right] \\ & -\frac{1}{192}\int_{M_{11}} \sqrt{-\hat{g}}\; d^{11}x \overline{\hat{\Psi}}_M \hat{\Gamma}^{MNPQRS} \hat{\Psi}_N (\hat{F}_4)_{PQRS} - \frac{1}{2}\int_{M_{11}} \hat{F}_4 \wedge *\hat{C}_4 \\ & -\frac{1}{12}\int_{M_{11}} \hat{F}_4 \wedge \hat{F}_4 \wedge \hat{A}_3 \ , \end{aligned} \tag{2.1}$$

where we have set the eleven-dimensional Newton's constant to unity and denoted

$$(\hat{C}_4)_{MNPQ} = 3\overline{\hat{\Psi}}_{[M} \hat{\Gamma}_{NP} \hat{\Psi}_{Q]} \ . \tag{2.2}$$

The spinor conjugation is defined to be $\overline{\hat{\Psi}}_M = \hat{\Psi}_M^\dagger \Gamma^0$. We have ignored the four-fermionic terms, which play no role in our analysis, and kept only bilinear terms in the gravitino field $\hat{\Psi}_M$. The action (2.1) is invariant under the usual supersymmetry transformations. For the gravitino this takes the form

$$\delta\Psi_M = D_M \epsilon + \frac{1}{288}\left(\hat{\Gamma}_M{}^{NPQR} - 8\delta_M^N \hat{\Gamma}^{NPQ}\right)(\hat{F}_4)_{NPQR}\, \epsilon \, , \tag{2.3}$$

where ϵ is the Majorana spinor parameterising the supersymmetry transformation. The conventions for the Dirac matrices Γ_M are given in appendix A.

One of the aims of this paper is to compactify the action (2.1) to give a four-dimensional theory. We achieve this by splitting the eleven-dimensional space as

$$M_{11} = M_4 \times K_7 \, . \tag{2.4}$$

As a consequence the metric in eleven dimensions decomposes into a direct product of a Minkowski signature metric on the four-dimensional space M_4 and an Euclidean one on the internal manifold K_7

$$ds_{11}^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu + \hat{g}_{mn} dy^m dy^n \, , \tag{2.5}$$

where $\mu, \nu = 0, \ldots, 3$ are curved indices for the four-dimensional Minkowski space and $m, n, \ldots = 4, \ldots 10$ are curved indices over the seven-dimensional compact space.

In order to compute fermionic terms in the four-dimensional effective action one needs to specify how the gamma matrices $\hat{\Gamma}_M$ decompose under (2.4)

$$\begin{aligned} \hat{\Gamma}_\mu &= \hat{\gamma}_\mu \otimes \mathbb{1} \, , \\ \hat{\Gamma}_m &= \gamma \otimes \hat{\gamma}_m \, . \end{aligned} \tag{2.6}$$

We deal with the conventions for $\{\hat{\gamma}_\mu\}, \{\hat{\gamma}_m\}$ in appendix A. The eleven-dimensional spinors decompose accordingly into a direct product of a four-dimensional spinor and a seven-dimensional one. As we will assume that the internal manifold K_7 has G_2 structure, we will mostly be interested in the cases in which the internal spinor is the globally defined spinor on such manifolds η (which we also choose to be a Majorana spinor)

$$\hat{\Psi}_M = (\hat{\psi}_M + \hat{\psi}_M^*) \otimes \eta \, . \tag{2.7}$$

$\hat{\psi}_M$ is taken to be a Weyl spinor in order to agree with the conventions which are mostly used in $N = 1$ supergravity [57] and hence we had to make the first term of the right hand side explicitly a Majorana spinor. The field $\hat{\psi}_\mu$ in the above decomposition will be the four-dimensional gravitino, while $\hat{\psi}_m$ will be a spin 1/2 field.

Finally we note that in order to have the $N = 1$ supergravity properly normalised one has to further perform field redefinitions such as the following:

$$\begin{aligned} \hat{g}_{\mu\nu} &= \mathcal{V}^{-1} g_{\mu\nu} \, , \\ \hat{\gamma}_\mu &= \mathcal{V}^{-1/2} \gamma_\mu \, , \\ \hat{\psi}_\mu &= \mathcal{V}^{-1/4} \psi_\mu \, . \end{aligned} \tag{2.8}$$

Note that the distinction between hatted and unhatted variables on the internal manifold is not particularly important, and we will generally omit hats where possible.

2.2 G_2-structures

For the following discussion it will be useful to point out a couple of aspects of manifolds with G_2 structure. A more systematic approach can be found in appendix B.

As we mentioned before, a manifold with G_2 structure admits a globally defined spinor, η. This spinor fails in general to be covariantly constant with respect to the Levi–Civita connection, but one can always find a connection with torsion such that

$$\nabla^{(T)}_m \eta := \nabla_m \eta - \frac{1}{4}\kappa_{mnp}\gamma^{np}\eta = 0 \; . \tag{2.9}$$

The tensor κ_{mnp} is the contorsion and gives a characterisation of the G_2 structure as explained in appendix B. Equivalently the G_2 structure can be described in terms of the invariant three-form φ

$$\varphi_{mnp} := i\overline{\eta}\gamma_{mnp}\eta \; , \tag{2.10}$$

and its Hodge dual

$$\Phi_{mnpq} := (*\varphi)_{mnpq} = -\overline{\eta}\gamma_{mnpq}\eta \; . \tag{2.11}$$

Throughout this paper, we will use the properties of these forms as set out in appendix B. For what follows it will be useful to note that due to (2.9) the forms φ and Φ are also covariantly constant with respect to the same connection with torsion. Mathematically this means

$$\begin{aligned} \nabla^{(T)}_m \varphi_{npq} &:= \nabla_m \varphi_{npq} - 3\kappa_{m[n}{}^{r}\varphi_{pq]r} = 0 \; , \\ \nabla^{(T)}_m \Phi_{npqr} &:= \nabla_m \Phi_{npqr} + 4\kappa_{m[n}{}^{s}\Phi_{pqr]s} = 0 \; . \end{aligned} \tag{2.12}$$

This formula is quite important as it allows one to evaluate the covariant derivative of these forms with respect to the Levi–Civita connection in terms of the (con)torsion.

Among the different G_2 structures of a special role in our analysis will be the so called manifolds with weak G_2 holonomy. They are characterised by the following relations (see also appendix C for more details)

$$\begin{aligned} d\varphi &= \tau * \varphi \\ d * \varphi &= 0 \; . \end{aligned} \tag{2.13}$$

The quantity τ is a constant on the manifold with G_2 structure which characterises completely the torsion of this type of manifolds. In the context of M-theory compactifications, as we shall see later in this paper, the parameter τ is not completely constant, but it can depend on the space-time coordinates through the volume of the internal manifold.

3 Superpotential in M-theory compactifications on manifolds with G_2 structure

Having described the general setup we can go and study in more detail M-theory compactifications on manifolds with G_2 structure. In this section we shall perform a general analysis which is valid for any manifold with G_2 structure, and obtain information about the Kähler potential and superpotential which appears in such compactifications.

3.1 Gravitino mass term

In compactifying string and M-theories down to four dimensions it is usually easier to derive the bosonic terms in the lower-dimensional action. Computing fermionic terms is often more involved and the fermionic part of the action is in general inferred from supersymmetry. There is, however, a specific class of terms which are relatively easy to compute and which give valuable information about the low energy effective action. These terms are the gravitino mass terms, and in $N = 1$ supergravity in four dimensions they take the form

$$M_{3/2} = \frac{1}{2} e^{K/2} \Big(\overline{W} \psi_\mu^T \gamma^{\mu\nu} \psi_\nu + W \overline{\psi}_\mu \gamma^{\mu\nu} \psi_\nu^* \Big) \ . \tag{3.1}$$

K here denotes the Kähler potential, which gives the coupling of the chiral fields to supergravity, and W is the superpotential. So by computing such gravitino mass terms one can obtain information about the Kähler potential and the superpotential. Similar calculations were performed in [58, 23, 49].[2]

Before we start the actual calculation, two comments about the gravitino are in order. Firstly, ψ_μ is taken to be a Weyl spinor to be consistent with (2.7), and represents the general result that any Majorana spinor χ can be expressed in terms of a Weyl spinor ψ by writing $\chi = \psi + \psi^c$ (although in our conventions, charge conjugation ^{c} is equivalent to complex conjugation *). Taking ψ_μ as positive chirality gives ψ_μ^* as negative chirality.

Secondly, as we explained before, the relation (2.7) yields both the four dimensional gravitino ψ_μ and also spin-1/2 fields ψ_m. Clearly, from the kinetic term in eleven dimensions one obtains a kinetic term for the gravitino, one for the spin 1/2 fields and one mixed kinetic term between the gravitino and the spin 1/2 fields. In the standard four-dimensional supergravity, such mixed terms are not present and to obtain the correctly normalised fermionic fields one has to perform a further redefinition of the gravitino field which in terms of the eleven dimensional field Ψ_M takes the form

$$\Psi_\mu \to \Psi'_\mu = \Psi_\mu + \delta\Psi_\mu \ , \quad \delta\Psi_\mu \sim \Gamma_\mu \Gamma^m \Psi_m \ . \tag{3.2}$$

We note that, since ψ_μ in the above relation appears linearly, the gravitino mass term (3.1) for ψ'_μ—when written in terms of the uncorrected gravitino field ψ_μ—has the same form up to terms which are linear in the field ψ_μ. Thus, computing the terms $\psi_\mu \gamma^{\mu\nu} \psi_\nu$, one can still deduce the combination $e^{K/2} W$ and so in order to make the calculation clearer we shall not be concerned with using the correct definition for the gravitino field (3.2).

We can now start to analyse the terms that contribute to the gravitino mass term in four dimensions. We shall have in mind the most general background compatible with Lorentz invariance in four dimensions, which includes internal fluxes F_{mnpq} and a flux in the four space-time dimensions $F_{\mu\nu\rho\sigma}$. Note that the background value for the fermionic fields is taken to vanish and so the four-fermionic terms in the eleven-dimensional action cannot contribute to the gravitino mass term in four dimensions. Therefore, we only need to consider the terms that we have kept in the action (2.1), which are bilinear in the gravitino field.

In the following we shall analyse one by one these contributions to the gravitino mass term.

[2] The same information can be obtained by reducing the supersymmetry transformations as was done for example in [59].

3.1.1 Contribution from the kinetic term

As also remarked in [49] the kinetic term for the gravitino only produces a contribution to the mass term in the presence of non-trivial structures, which will be proportional to $D_m\eta$. Inserting the decomposition (2.7) in the gravitino kinetic term from (2.1) and keeping only the terms which are relevant for the gravitino mass term one finds

$$\begin{aligned}\overline{\hat{\Psi}}_M\hat{\Gamma}^{MNP}D_N\hat{\Psi}_P &= \overline{\hat{\Psi}}_\mu\hat{\Gamma}^{\mu n\nu}D_n\hat{\Psi}_\nu + \text{ terms not contributing to gravitino mass} \\ &= -(\overline{\hat{\psi}}_\mu + \overline{\hat{\psi}^*}_\mu)\hat{\gamma}^{\mu\nu}\gamma(\hat{\psi}_\nu + \hat{\psi}^*_\nu)\eta^T\gamma^n D_n\eta + \dots \end{aligned} \tag{3.3}$$

We compute the covariant derivative on the spinor η by making use of (2.9) and we find that

$$\eta^T\gamma^n D_n\eta = -\frac{i}{4}\varphi^{mnp}\kappa_{mnp} \ , \tag{3.4}$$

where κ_{mnp} denotes the intrinsic contorsion and the G_2 three-form φ is defined in terms of the spinor η in (2.10). Note that this quantity picks up the singlet piece under G_2 of the intrinsic contorsion and can be written using the formulae in appendix B as

$$\varphi^{mnp}\kappa_{mnp} * \mathbf{1} = \frac{1}{2}d\varphi\wedge\varphi \ . \tag{3.5}$$

Taking into account the rescalings in (2.8), the contribution to the gravitino mass coming from the kinetic term of the gravitino in eleven dimensions can be written

$$M_{3/2}^{(\text{k.t.})} = \left(\frac{i}{16\mathcal{V}^{3/2}}\int d\varphi\wedge\varphi\right)\,\overline{\psi}_\mu\gamma^{\mu\nu}\psi^*_\nu + \text{c.c.} \tag{3.6}$$

3.1.2 Contribution from the internal flux

The next contribution to the gravitino mass term we discuss is the one that appears due to the internal fluxes. Note that if one takes into account a non-trivial G_2 structure the internal flux also receives contribution from the non-closure of the forms in which we expand the eleven-dimensional fields (see for example [49]). We will elaborate on this issue later and for now we denote the internal fluxes by $\hat{F}_4$ and do not discuss their origin here. The relevant term can be written

$$\begin{aligned}\overline{\hat{\Psi}}_M\hat{\Gamma}^{MNPQRS}\hat{\Psi}_N(\hat{F}_4)_{PQRS} &= \overline{\hat{\Psi}}_\mu\Gamma^{\mu\nu pqrs}\hat{\Psi}_\nu(\hat{F}_4)_{pqrs} + \dots \\ &= (\overline{\hat{\psi}}_\mu + \overline{\hat{\psi}^*}_\mu)\hat{\gamma}^{\mu\nu}(\hat{\psi}_\nu + \hat{\psi}^*_\nu)\eta^T\gamma^{pqrs}\eta(\hat{F}_4)_{pqrs} + \dots \end{aligned} \tag{3.7}$$

Using (2.11) to eliminate the $\eta^T\gamma^{pqrs}\eta$, taking into account all the factors in the action and the rescalings from (2.8), the final result is

$$M_{3/2}^{(\text{i.f.})} = \left(\frac{1}{8\mathcal{V}^{3/2}}\int \hat{F}_4\wedge\varphi\right)\,\overline{\psi}_\mu\gamma^{\mu\nu}\psi^*_\nu + \text{c.c.} \tag{3.8}$$

3.1.3 Flux along the space-time directions

Having a flux for $\hat{F}_4$ completely in the four space-time directions can also generate a mass term for the gravitino from the last term in the second line of (2.1). After all rescalings are taken into account, this would be simply given by

$$M_{3/2}^{(\text{e.f.})} = -\frac{1}{2}\mathcal{V}^{3/2}dA_3\wedge *C_4 \ , \tag{3.9}$$

where C_4 is defined as

$$(C_4)_{\mu\nu\rho\sigma} = 3(\overline{\psi} + \overline{\psi^*})_{[\mu}\gamma_{\nu\rho}(\psi + \psi^*)_{\sigma]} = \frac{i}{4}\left(\bar{\psi}_\lambda \gamma^{\lambda\tau}\psi^*_\tau\right)\epsilon_{\mu\nu\rho\sigma} + \text{c.c.} \tag{3.10}$$

As we will see in a short while, however, a purely four-dimensional flux for $\hat{F}_4$ is not essential for obtaining some contribution for the gravitino mass term from the above expression. The reason is that we would have to dualise the three-form A_3 in a consistent way. A three-form in four dimensions is not dynamical and its dual is thus only an arbitrary constant [60]. It is important to stress that even if one chooses this constant to vanish the dualisation of A_3 produces in general a non-trivial result due to its couplings in the four-dimensional action. Thus, in order to obtain the correct result we would have to derive first the complete action for the four-dimensional field A_3. This requires that we make a specific Ansatz for the decomposition of the eleven-dimensional field $\hat{A}_3$ in terms of four-dimensional fields. At this stage we know almost nothing about the correct way of performing such an expansion and so consider

$$\hat{A}_3 = \mathring{A}_3 + A_3 + a_3 \ , \tag{3.11}$$

where $\mathring{A}_3$ is the background value for $\hat{A}_3$, which gives rise to the background flux, $G := d\mathring{A}_3$, while A_3 and a_3 represent fluctuations around it. A_3 is the four-dimensional three-form field and a_3 is a three-form which lives in the internal manifold, but which depends on the space-time as well. From the point of view of the low energy action this form, a_3, will produce scalar fields in four dimensions. Note that in general one can consider also fluctuations which from the four-dimensional perspective are two-forms or vector fields, but as these other fields play no role in the following discussion we ignore them completely. However, it is important to notice that apart from these other degrees of freedom, (3.11) is the most general expression one can consider. Once a specific model is chosen the above Ansatz can be further refined (see e.g. section 4).

Inserting (3.11) into (2.1) and also considering the rescalings (2.8) one can derive the terms in the low energy action which contain A_3. One is the kinetic term, and it is easy to see that this has the form

$$-\frac{\mathcal{V}^3}{4} dA_3 \wedge *dA_3 \ . \tag{3.12}$$

The second contribution comes from the Chern-Simons term and is present only if the internal value for the field $\hat{F}_4$ is non-zero. As noted before this receives in general contributions from two sources: from fluxes and from non-trivial structures. To make the calculation clearer we will consider each case in turn.

Case I: fluxes on a manifold with G_2 holonomy

On a manifold with G_2 holonomy the massless scalar fields which come from $\hat{A}_3$ appear from the expansion in harmonic three-forms. Translated into our notation it means that we have to take a_3 to be harmonic on the internal manifold. Turning on fluxes in this case would mean that $\hat{F}_4$ has the following expansion

$$\hat{F}_4 = G + d_4 A_3 + d_4 a_3 \ , \tag{3.13}$$

where the index 4 on the exterior derivatives shows that they are taken in the four space-time directions. G is the four-form background flux as defined above. It is not hard to see now that the above expansion leads to the following expression for the Chern–Simons coupling

$$\int_{K_7} \hat{F}_4 \wedge \hat{F}_4 \wedge \hat{A}_3 = 6\left(\int_{K_7} G \wedge a_3\right) d_4 A_3 \ . \tag{3.14}$$

Case II: non-trivial G_2 structure and no fluxes

As mentioned before, even if no fluxes are turned on one can in general expect purely internal values for the field strength $\hat{F}_4$ from terms like $d_7 a_3$. With this in mind, the expansion of the field strength $\hat{F}_4$ takes the form

$$\hat{F}_4 = d_4 A_3 + d_4 a_3 + d_7 a_3 \ . \tag{3.15}$$

The computation of the Chern–Simons term can now be seen to yield

$$\int_{K_7} \hat{F}_4 \wedge \hat{F}_4 \wedge \hat{A}_3 = 3 \left(\int_{K_7} d_7 a_3 \wedge a_3 \right) \ d_4 A_3 \ , \tag{3.16}$$

As the above contributions to the action for A_3—(3.16) and (3.14)—are linear in G and $d_7 a_3$, it is clear that in the case where one turns on non-trivial fluxes in a compactification on some manifold with G_2 structure the total contribution to the term proportional to dA_3 in the four-dimensional action is just the sum of the above contributions. Taking into account the original term, (3.9), the complete action for the field A_3 in four dimensions reads

$$-\frac{\mathcal{V}^3}{4} dA_3 \wedge *dA_3 - \frac{\mathcal{V}^{3/2}}{2} dA_3 \wedge *C_4 - \frac{1}{2} dA_3 \int_{K_7} \left(G + \frac{1}{2} d_7 a_3 \right) \wedge a_3 \ . \tag{3.17}$$

Denoting the constant which is dual to A_3 by λ the dual action takes the form [60] (see also appendix E.2 of [61])

$$-\frac{1}{4\mathcal{V}^3} \left[\lambda - \int_{K_7} \left(G + \frac{1}{2} d_7 a_3 \right) \wedge a_3 \right]^2 * \mathbf{1} + \frac{1}{2} \left[\lambda - \int_{K_7} \left(G + \frac{1}{2} d_7 a_3 \right) \wedge a_3 \right] \mathcal{V}^{3/2} C_4 \ , \tag{3.18}$$

where we have neglected again four-fermionic terms. The last term in the above expression is now truly the term which contributes to the gravitino mass term.

3.2 The superpotential

We are now in position to compute the superpotential which appears in M-theory compactifications on seven-dimensional manifolds with G_2 structure. Putting together the contributions to the gravitino mass term from (3.6), (3.8) and (3.18), noting that the flux $\hat{F}_4$ in (3.8) is in fact $G + d_7 a_3$, one finds

$$M_{3/2} = -\frac{i}{16\mathcal{V}^{3/2}} \bar{\psi}_\mu \gamma^{\mu\nu} \psi^*_\nu \left[\int (da_3 + id\varphi) \wedge (a_3 + i\varphi) + 2\lambda + 2 \int G \wedge (a_3 + i\varphi) \right] + \text{c.c.} \tag{3.19}$$

Using (3.1) one immediately obtains (up to an overall phase which plays no role in the definition of the superpotential)

$$e^{K/2} W = \frac{1}{8\mathcal{V}^{3/2}} \left[\int (da_3 + id\varphi) \wedge (a_3 + i\varphi) + 2\lambda + 2 \int G \wedge (a_3 + i\varphi) \right] + \text{c.c.} \tag{3.20}$$

This is the first main result of our paper so let us pause to discuss it for a while. First of all we note that in $N = 1$ supergravity, the Kähler potential and the superpotential are not truly independent functions, but the only thing which has a physical meaning is the combination $e^K |W|^2$. Obviously the equation above gives us this quantity. However we can not resist noting that this formula looks

very suggestive and that it splits up in a natural way in a part which is holomorphic (the part in the brackets) and an overall real factor. It is thus tempting to argue that for general compactifications on manifolds with G_2 structure the Kähler potential for the low energy fields is always given by the same formula as in the case of G_2 holonomy

$$K = -3 \ln \mathcal{V} \, . \tag{3.21}$$

Establishing this, the superpotential is then given by

$$W = \frac{1}{8} \left[\int (d\varphi + i da_3) \wedge (\varphi + i a_3) + 2\lambda + 2 \int G \wedge (\varphi + i a_3) \right] \, . \tag{3.22}$$

One immediately observes in this superpotential that the last term is precisely the result obtained in [7] for the case of manifolds with G_2 holonomy. The second term, which is just a constant, should always be present and appears from the correct dualisation of the field A_3 in four dimensions. It also appears in [7], but as a quantum effect and in that case it turned out to be quantised. Here we only limit ourselves to the supergravity approximation and thus we keep this additional parameter real. Finally one notices the first term which appears entirely due to the non-trivial G_2 structure (intrinsic torsion). This term is completely new, and in the next section we will compute it explicitly for the case of manifolds with weak G_2 holonomy and also show that it reproduces the potential which can be derived from the dimensional reduction when inserted in the usual $N = 1$ formula.

Before we end this section we make one more comment on the way this superpotential was derived. In the original action for M-theory (2.1), the fermion bilinears coupled only linearly to the Field strength $\hat{F}_4$. Naively one would conclude from this that the superpotential can depend only linearly on the flux G or the fields which appear in the low energy spectrum from $\hat{A}_3$. However it is clear that the superpotential (3.22) contains also quadratic terms like $G \wedge a_3$ or $da_3 \wedge a_3$. Tracing back these terms we see that they only appear from the correct dualisation of the three form A_3 in four dimensions (3.18). The observation we want to make is that there is a simpler and more direct way to see such terms appearing by considering the starting action to be the 'duality symmetric' M-theory action [55]. In such a formulation of M-theory one also has a six-form field $\hat{A}_6$ with seven-form field strength which is defined like

$$\hat{F}_7 = d\hat{A}_4 + A_3 \wedge F_4 \, . \tag{3.23}$$

The fermionic action will now contain fermion bilinears which also couple to $\hat{F}_7$ and from such couplings and also the second term in the definition of $\hat{F}_7$ one can immediately see quadratic terms in $\hat{A}_3$ appearing in the superpotential.

4 Metric deformation space of manifolds with weak G_2 holonomy

So far we have only discussed general features of M-theory compactifications on manifolds with G_2-structure without making any reference to the low-energy field content of such theories. In order to be able to obtain a specific model in four dimensions one needs to have some more information about the internal properties of such manifolds and in particular one needs to get a handle on their moduli space.[3] In general this question is quite complicated and a satisfactory answer has

[3]Strictly speaking the word modulus/moduli stand for fields which are exactly flat directions of the potential. What we actually mean by moduli in this section and also in the whole paper are the fields which appear from the metric deformations on the internal manifold. Generically, as we shall see in a while, such fields appear with a potential and it is precisely this feature we are interested to capture here, namely determine the potential for such fields which would enable one to perform a correct analysis of the field stabilisation in such models.

not been found yet. For special cases, however, such as the one we will discuss here, it turns out to be possible to gain information about the space of deformations of these manifolds and using this information to compute the low-energy action in four dimensions. It will be our purpose in this section to solve some of the problems outlined here for the case of manifolds with weak G_2 holonomy.

4.1 Deformations of the weak G_2 structure

In this section and also in the following ones we will use many properties of manifolds with G_2 structures. For the reader who is not familiar with these notions we have assembled a couple of appendices where we present the relations we will use in these sections. We will only outline the key facts about the deformation space of manifolds with weak G_2 holonomy and prove many of the underlying (rather important) facts in the appendix C.

We start by noting that specifying a G_2 structure φ on a seven-dimensional manifold uniquely determines the metric. The relevant equation for this is (B.20), but for our purposes it suffices to note that the variations of the metric can be encoded in variations of the G_2 three-form φ. Thus, as in the case of G_2 holonomy, the deformations of metrics on manifolds with G_2 structure can be studied by looking at variations of the invariant form φ. For a general G_2 structure the situation is a bit complicated because the torsion itself can vary together with the structure. For this reason, from now on we shall concentrate on the special class of manifolds with weak G_2 holonomy (2.13) (for a detailed description see appendix C). From (2.13) it is clear that the torsion τ cannot depend explicitly on the coordinates of the internal manifold, but from what was said above, it can in principle depend on its moduli and as we shall see later, in our case it does.

The strategy we adopt is the following. We consider first a G_2 structure φ which satisfies the weak G_2 conditions (2.13). Then we consider small variations of this G_2 structure by some arbitrary form $\delta\varphi$ and impose that the equations (2.13) are satisfied for some $\tau' = \tau + \delta\tau$. This will yield some conditions on the variations $\delta\varphi$ and $\delta\tau$. It is important to stress here that by changing φ, the metric on the manifolds changes as well and thus the definition of the Hodge star changes. In order not to create any confusion we will use the notations from [15]. Proposition 10.3.5 from [15] gives the form of the Hodge dual of a perturbed G_2 structure to be

$$\Theta(\varphi + \delta\varphi) = *\varphi + \frac{4}{3} * P_1\delta\varphi + *P_7\delta\varphi - *P_{27}\delta\varphi \, , \tag{4.1}$$

where $\Theta(\xi)$ is a map $\Theta : \Lambda^3 \to \Lambda^4$ which computes the Hodge dual of a three-form ξ with the metric defined by ξ via (B.20). The Hodge star on the right hand side of (4.1) is defined from the old (unvaried) metric are P_1, P_7 and P_{27}, which denote projection operators on the spaces of corresponding dimensionality. Note that on the space of three-forms $P_1 + P_7 + P_{27} = \mathbb{1}$.

Imposing the first relation in (2.13) for the varied form $\varphi + \delta\varphi$ at the first order in the perturbations we can write

$$d\delta\varphi = \delta\tau * \varphi + \tau \left(\frac{4}{3} * P_1\delta\varphi + *P_7\delta\varphi - *P_{27}\delta\varphi \right) \, . \tag{4.2}$$

Thus, perturbing a weak G_2 structure φ by some form $\delta\varphi$ leads again to a weak G_2 structure provided the variation $\delta\varphi$ satisfies (4.2) for some suitable $\delta\tau$.

At this stage, we make use of the observation that the **7** component of $\delta\varphi$ makes no contribution to perturbations of the induced metric through the formula (B.22). We shall therefore set such

components to zero, which will simplify the analysis below. It is shown in the Appendix that for forms which satisfy $P_7\delta\varphi = 0$, the projectors P_1 and P_{27} commute with the exterior derivative. Since these projectors also commute with the Hodge star one can break (4.2) into two simpler conditions for the singlet variations $P_1\delta\varphi$ and for the ones which transform as a **27** under G_2

$$\begin{aligned} dP_1\delta\varphi &= \delta\tau * \varphi + \frac{4}{3}\tau * P_1\delta\varphi \, , \\ dP_{27}\delta\varphi &= -\tau * P_{27}\delta\varphi \, . \end{aligned} \tag{4.3}$$

From here it is clear that the torsion τ depends on the deformations of the weak G_2 manifold only via the singlet deformation $P_1\delta\varphi$. In other words, since such singlet deformations only rescale the G_2 structure φ and through it the volume of the manifold, we conclude that the torsion τ depends on the parameters describing the weak G_2 manifold only via its volume.

Let us now consider the two equations above separately. We start with the first one and parameterise the singlet part of the deformation as

$$P_1\delta\varphi = \epsilon\varphi \, , \tag{4.4}$$

for some arbitrary small ϵ. Then equation (4.3) becomes

$$\epsilon d\varphi = \delta\tau * \varphi + \frac{4}{3}\tau\epsilon * \varphi \, . \tag{4.5}$$

Using again the weak G_2 condition (C.4) we obtain

$$\delta\tau = -\frac{\epsilon}{3}\tau \, . \tag{4.6}$$

From the definition of the volume in terms of the G_2 structure φ (B.6) and (4.1), it is not hard to see that under the deformation (4.4) the volume changes as

$$\delta\mathcal{V} = \frac{7}{3}\epsilon\mathcal{V} \, . \tag{4.7}$$

Dividing the last two equations we obtain that the variation of the torsion τ with the volume obeys

$$\frac{\delta\tau}{\tau} = -\frac{1}{7}\frac{\delta\mathcal{V}}{\mathcal{V}} \, , \tag{4.8}$$

or after integration

$$\tau \sim \mathcal{V}^{-1/7} \, . \tag{4.9}$$

Intuitively the above equation can be understood as follows. As mentioned before singlet variations rescale the G_2 structure φ. In its turn, such a rescaling produces a rescaling of the metric via (B.22) and so the scalar curvature R is rescaled. Since the torsion is directly related to the scalar curvature via (C.10) it follows that such a deformation can only be present if the torsion of the manifold itself changes and the quantitative measure of this change is captured by the above equation.

Let us now turn our attention to the second equation in (4.3). As we said before, in the case of variations with forms which transform as **27** under G_2 the torsion does not vary. Intuitively, since the torsion is a singlet under G_2 we would need another object which transforms non-trivially under G_2 in order to produce a singlet out of the deformation $\delta\varphi$. Since we do not have at our disposal any such thing it can be understood that the torsion can not change under such deformations. Thus,

the second equation in (4.3) only imposes a condition on the variations of φ which are compatible with the weak G_2 structure.

So far we have learned that the non-trivial metric deformations of weak G_2 manifolds are parameterised by three-forms $\delta\varphi$ satisfying

$$\begin{aligned} P_7\delta\varphi &= 0 \ , \\ dP_{27}\delta\varphi &= - * P_{27}\delta\varphi \ . \end{aligned} \tag{4.10}$$

Note the counter-intuitive minus sign appearing on the right hand side which is going to be crucial in determining the correct mass of the modes associated with these variations of the G_2 structure.

Let us now try to give a more explicit parameterisation of the deformation space. Recall that for the case of manifolds with G_2 holonomy the form φ was closed and coclosed and thus harmonic. Consequently one expanded this form in a basis for harmonic three-forms with the coefficients being the moduli fields. Then by general methods, which we present in appendix B, one could compute the metric on the moduli space. Let us try to do something similar here. Clearly from the relations (2.13) we see that the form φ cannot be harmonic anymore and in fact the torsion τ measures its failure to be harmonic. However, it is easy to compute the action of the Laplace operator on φ and one obtains

$$\Delta\varphi \equiv (*d * d + d * d*)\varphi = \tau^2\varphi \ , \tag{4.11}$$

and thus φ is still an eigen-form of the Laplace operator corresponding to the eigenvalue τ^2. Consider a basis Π_i for the three-forms satisfying

$$\Delta\Pi_i \equiv (*d * d + d * d*)\Pi_i = \tau^2\Pi_i \ , \tag{4.12}$$

where these forms do not depend on the moduli of the weak G_2 manifold. As it stands this assumption is definitely too strong. In fact what happens for harmonic forms is that their dependence on the moduli comes in only via exact forms. Assuming the same thing happens also in our case, we show in the appendix that the calculation which follows from here on is indeed consistent. For now we can expand the G_2 structure φ in this basis

$$\varphi = s^i\Pi_i \ , \tag{4.13}$$

and as for manifolds of G_2 holonomy we think of the forms Π_i as independent of the choice of metric. For the parameters s^i to be truly moduli of the weak G_2 structure, we still need that the forms Π_i satisfy the condition (4.10). From here on we will assume that the forms Π_i used in the expansion (4.13) do satisfy this condition as well. If this is true, than the coefficients s^i are indeed the scalar fields which characterise the possible deformations of the weak G_2 manifold.

As one does on a manifold with G_2 holonomy, let us further define

$$\mathcal{K}_i = \int \Pi_i \wedge *\varphi \ . \tag{4.14}$$

Using the fact that the forms Π_i satisfy (4.10) one derives

$$\begin{aligned} d\Pi_i &= d(P_1\Pi_i + P_{27}\Pi_i) \\ &= dP_1\Pi_i - \tau * P_{27}\Pi_i \\ &= -\tau * \Pi_i + 2dP_1\Pi_i \ . \end{aligned} \tag{4.15}$$

The projector on the singlet subspace P_1 is defined by

$$P_1\Pi_i = \frac{1}{7\mathcal{V}}\int \Pi_i \wedge *\varphi = \frac{\mathcal{K}_i}{7\mathcal{V}} \; , \tag{4.16}$$

where we have used that $(\Pi_i)_{mnp}\varphi^{mnp}$ does not depend on the internal manifold, fact which is also proven in the appendix. Using the weak G_2 conditions (2.13) one immediately finds

$$d\Pi_i = -\tau * \Pi_i + \frac{2\tau\mathcal{K}_i}{7\mathcal{V}} * \varphi \; . \tag{4.17}$$

This is the central relation of this second part of the paper as it will allow us to compute explicitly the low-energy effective action for compactifications on manifolds with weak G_2 holonomy and compare it with the result derived before on general grounds.

It is important to stress one more thing here. At this stage we cannot say much more about the space of deformations of weak G_2 manifolds, apart from the fact that it can be characterised in terms of the forms Π_i satisfying (4.10), which will be exploited in the next section. One thing is, however, quite important. Note that the operator $\Delta - \tau^2$ is an elliptic operator. It is known in the general theory of operators that elliptic operators on compact manifolds have a finite dimensional kernel. This means that at least the space of deformations of weak G_2 manifolds is *finite*.[4]

Before we move on let us summarise the main results that we derived in this section. For weak G_2 manifolds the invariant form φ turns out to be an eigenform of the Laplace operator (4.11). Performing the expansion in terms of the deformations in the same way as one did on manifolds with G_2 holonomy we conclude that the forms in which we perform this expansion must obey (4.10). It is interesting to note that this relation was obtained in a pretty generic fashion. Similar relations were derived for half-flat manifolds [41] but using mirror symmetry as an additional source of information for the manifolds with $SU(3)$ structure. Moreover those relations were valid only in a certain limit which was denoted as the small torsion limit while the relations above are valid for any torsion. We will see later on that the torsion cannot be small as it turns out to be of the order of the inverse radius of the manifold.

4.2 Useful formulae on the deformation space of weak G_2 manifolds

Before we perform the compactification we will find it useful to derive some formulae which make the calculation on the deformation space of weak G_2 manifolds easier.

The presence of the forms Π_i which are not closed (although they are co-closed) allows us to introduce a topological, two-index, symmetric object on these manifolds

$$\mathcal{K}_{ij} = \int \Pi_i \wedge d\Pi_j = \mathcal{K}_{ji} \; . \tag{4.18}$$

Obviously, the appearance of such a matrix is only due to the non-minimal structure as it depends on $d\Pi_i$, which would clearly vanish for the case of manifolds with G_2 holonomy. As we shall see later on this object will enter the expression of the superpotential in terms of the low energy fields.

A straightforward calculation, which we have outlined in the appendix, shows that for a general expansion of the form (4.13) the sigma model metric for the moduli takes the form

$$g_{ij} = \frac{1}{4\mathcal{V}}\int \Pi_i \wedge *\Pi_j \; . \tag{4.19}$$

[4] We thank André Lukas for drawing our attention on this fact.

Using (4.17) and (4.18) it is easy to show that

$$g_{ij} = -\frac{1}{4\tau\mathcal{V}}\mathcal{K}_{ij} + \frac{\mathcal{K}_i\mathcal{K}_j}{14\mathcal{V}^2} \ . \tag{4.20}$$

Furthermore one also has the usual relations

$$\begin{aligned} \mathcal{K}_i s^i &= 7\mathcal{V} \ , \\ \mathcal{K}_i &= 4\mathcal{V} g_{ij} s^j \ , \\ \mathcal{K}_i g^{ij} &= 4\mathcal{V} s^j \ . \end{aligned} \tag{4.21}$$

The matrix $\mathcal{K}_{ij}$ introduced in (4.18) can be shown to satisfy

$$\begin{aligned} &\mathcal{K}_{ij} s^j = \tau \mathcal{K}_i \ , \\ &\mathcal{K}_{ij} g^{jk} = -4\tau\mathcal{V}\delta_i^k + \frac{8}{7}\tau\mathcal{K}_i s^k \ . \end{aligned} \tag{4.22}$$

Using these relations we can now proceed and compute the effective action which arises by compactifying M-theory on manifolds with weak G_2 holonomy.

5 M-theory compactifications on manifolds with weak G_2 holonomy

Having discussed in the previous section the possible deformations of weak G_2 manifolds we shall now move on and derive the low energy action which appears when compactifying M-theory on such manifolds. Then we will show that the resulting theory is an $N = 1$ supergravity coupled to chiral multiplets. The correspnding Kähler potential and superpotential will turn out to be the ones derived on general grounds in section 3.2.

5.1 Review of the Freund–Rubin solution

In order to understand properly the process of the compactification it is useful to review the Freund–Rubin reduction Ansatz which turns out to be the relevant setup for compactifications on manifolds with weak G_2 holonomy. Recall that in the Freund–Rubin background, only the flux along the four space-time directions is non-vanishing and constant and the space-time is considered to be maximally symmetric. The unique possibility for the background is then

$$F_{\mu\nu\rho\sigma} = \frac{3}{2}\tau\epsilon_{\mu\nu\rho\sigma} \ , \tag{5.1}$$

where the factor 3/2 is chosen for later convenience. We know that such a background is supersymmetric if the supersymmetric variation of the gravitino vanishes. Inserting (5.1) into (2.3) and splitting the eleven-dimensional spinor like in section 3, one immediately finds that for the internal manifold

$$\nabla_m\eta + \frac{i}{8}\tau\gamma_m\eta = 0 \ . \tag{5.2}$$

This is precisely the weak G_2 condition in terms of the spinor η (C.8) which was shown in the appendix C.1 to be equivalent to (2.13). We note that the most general seven-dimensional manifold

that produces a maximally symmetric space-time and preserves supersymmetry in the background (5.1) is a weak G_2 manifold[5] satisfying (5.2) or equivalently (2.13). Consequently looking for small deformations of this solution that still satisfy the equations of motion in the background (5.1) is equivalent to looking for small variations of the metric on the weak G_2 manifold that lead to another weak G_2 manifold with some torsion $\tau + \delta\tau$. One can see that the parameters s^i obtained from the deformations of the weak G_2 structure will indeed have the interpretation of scalar fields in four dimensions.

In general Kaluza–Klein compactifications one first identifies the massless modes and truncates away the massive towers of modes which appear. For this it is necessary to identify correctly the mass terms for the various fields in the theory.

For compactifications on G_2 manifolds, which are Ricci flat, this is a straightforward exercise and the masses of the different modes can be obtained by studying the spectrum of the Laplace operators acting on various degree forms and of the Lichnerowicz operator. For the case at hand the situation is a bit more complicated because of the fact that manifolds with weak G_2 holonomy have a non-vanishing Ricci curvature. This implies that in general the ground state of such a theory is no longer Minkowski, but AdS. In particular the Ricci tensors for the external and internal spaces are respectively

$$R_{\mu\nu} = -\frac{3}{4}\tau^2 g_{\mu\nu} \; , \qquad R_{mn} = \frac{3}{8}\tau^2 g_{mn} \; . \tag{5.3}$$

Note that the setup we are dealing with is significantly changed from what was considered before in the literature. The torsion in our case is finite and in particular related to the radius of the internal manifold. It is not arbitrary small as, for example, was previously assumed for half-flat manifolds [41]. Thus one has to take into account all the large effects coming from torsion and fluxes in order to determine the correct ground state. The general analysis for the compactification of M-theory on seven-dimensional Einstein manifolds was performed in [1], and so we will just adapt the formulæ they use for our own purposes.

In our analysis we will only be interested in the scalar fields which appear in the four-dimensional effective action. Such mass operators were computed for example in [1] and for the conventions we use in this paper they take the form

$$\begin{aligned} M^2_{0^-} =&\; Q^2 + \frac{3}{2}\tau Q + \frac{1}{2}\tau^2 = (Q+\tau)(Q+\frac{1}{2}\tau), \\ M^2_{0^+} =&\; \Delta_L - \frac{1}{4}\tau^2 \; , \end{aligned} \tag{5.4}$$

where we have defined as in [1] the operator $Q := *d$. The former of these operators is for three-form matter fields and the latter for traceless symmetric variations of the metric. For three-form structure variations that generate such metric variations via (B.22), this operator can—after a tedious calculation involving the Lichnerowicz operator Δ_L—be written

$$M^2_{0^+} = Q(Q + \frac{1}{2}\tau) \; . \tag{5.5}$$

The presence of supersymmetry should, however, complexify the G_2 structure φ by the matter three-form a_3 and therefore to preserve $N = 1$ supersymmetry in four dimensions one needs to expand a_3 in the same way as φ. It is not hard to see using the above formulae for the masses of the scalars coming from the metric deformations and A_3 matches the mass pattern of a Wess-Zumino (chiral) multiplet in AdS space [56], confirming our expectations about supersymmetry. Finally we note that the forms satisfying (4.10) turn out to be zero-modes of the operator $M^2_{0^-}$.

[5]For a rigorous proof of this statement see [4].

5.2 The compactification

To perform the compactification on such manifolds with weak G_2 holonomy one has first to identify the fields which appear in four dimensions. In the previous section we have argued that the AdS massless modes (scalars) which appear in compactifications on manifolds with weak G_2 holonomy are given by the expansion in forms which satisfy (4.10). Neglecting as in the usual Kaluza–Klein setups the rest of the massive towers of states we can now perform the compactification on weak G_2 manifolds and keep only the modes discussed above.

From the expansion (4.13) and the relations (B.20) one can derive what will be the kinetic term for the scalars s^i which comes from the expansion of the Ricci scalar. As in the case of manifolds with G_2 holonomy the sigma-model metric takes the form

$$g_{ij} = \frac{1}{4\mathcal{V}} \int \Pi_i \wedge *\Pi_j \ . \tag{5.6}$$

In the matter sector we perform a similar expansion to (4.13). In this paper we will only be interested in the scalar fields which arise in the compactification of M-theory on a manifold with weak G_2 holonomy. There can be also other fields like vectors, but here we will ignore them completely. If we denote again as in (3.11) the internal component of the field A_3 by a_3, then we write

$$a_3 = a^i \Pi_i \ . \tag{5.7}$$

The full eleven-dimensional three form $\hat{A}_3$ then takes the form

$$\hat{A}_3 = A_3 + a^i \Pi_i \ , \tag{5.8}$$

where A_3 is a three-form in four dimensions. This is not dynamical and so it can be dualised to a constant as we did in section 3. The four-dimensional bosonic action which one derives in this way has the form

$$S_4 = \frac{1}{2} \int \left[\sqrt{-g} R - g_{ij} dT^i \wedge *d\bar{T}^j - \sqrt{-g} V \right] \ . \tag{5.9}$$

The potential V comes from three distinct places. First it comes from the purely internal part of of $\hat{F}_4$. This will have the form

$$V_1 = \frac{1}{8\mathcal{V}^2} \int da_3 \wedge *da_3 \ , \tag{5.10}$$

where the exterior derivative is understood to be in the internal manifold direction and the factor $1/\mathcal{V}^2$ comes from the Weyl rescaling in four dimensions, (2.8). Using (4.17) this can be easily seen to be

$$V_1 = \frac{\tau^2}{8\mathcal{V}^2} a^i a^j \int \Pi_i \wedge *\Pi_j = \frac{\tau^2}{2\mathcal{V}} a^i a^j g_{ij} \ . \tag{5.11}$$

From the dualisation of A_3 in four dimensions we have already seen that there is a contribution to the potential (3.18). Using (5.7) and (4.17) one finds[6]

$$V_2 = \frac{1}{4\mathcal{V}^3} \left(\lambda - \frac{a^i a^j \mathcal{K}_{ij}}{2} \right)^2 \ . \tag{5.12}$$

Finally one has to take into account the contribution from the curvature of the internal manifold. The Ricci scalar of weak G_2 manifolds can be easily computed (C.10), and after performing the

[6]Note that we now take the dual of A_3 in four dimensions to be the constant λ, which can in principle be independent of the background value considered in (5.1).

integration over the internal manifold whilst taking into account the factor $1/\mathcal{V}^2$ coming from the Weyl rescaling in four dimensions one obtains

$$V_3 = -\frac{21\tau^2}{16\mathcal{V}} \ . \tag{5.13}$$

The potential coming from the compactification thus takes the form

$$\begin{aligned} V &= V_1 + V_2 + V_3 \\ &= \frac{1}{16}\left[-21\frac{\tau^2}{\mathcal{V}} + \frac{1}{\mathcal{V}^3}(a^i a^j \mathcal{K}_{ij})^2 + 16\frac{\tau^2}{\mathcal{V}} a^i a^j g_{ij}\right] \ . \end{aligned} \tag{5.14}$$

5.3 Comparison with the general result

To conclude this analysis we still have to show that the result obtained in the previous subsections is indeed an $N = 1$ supergravity. As we have neglected completely the gauge fields we only have to find the corresponding Kähler potential and superpotential. This is not a hard task since we have at our disposal the general result derived in section 3.2. In appendix B it was shown that the metric (5.6) is Kähler, i.e. $g_{ij} = \partial_i \partial_{\bar{j}} K$, and the Kähler potential is

$$K = -3 \ln \mathcal{V} \ . \tag{5.15}$$

As we argued in section 3.2, the superpotential is given by (3.22), which for the case where a non-trivial structure but no G fluxes are taken into account becomes

$$W = \frac{1}{8}\int_{M_7} d(a_3 + i\varphi) \wedge (a_3 + i\varphi) + \frac{\lambda}{2} \ . \tag{5.16}$$

Using the field expansions (4.13) and (5.7) and also the definition (4.18) we obtain the superpotential in terms of the four-dimensional fields

$$W = \frac{\mathcal{K}_{ij}}{8} T^i T^j + \frac{\lambda}{2} \ , \tag{5.17}$$

where the complex fields T^i are defined as

$$T^k = a^k + i s^k \ . \tag{5.18}$$

To show that the action (5.9) is the bosonic part of an $N = 1$ supergravity theory we have to show that the potential (5.14) can be derived from the superpotential (5.17) using the general supergravity formula

$$V = e^K \left[D_i W \overline{(D_j W)} g^{\bar{j}i} - 3|W|^2\right] \ , \tag{5.19}$$

where as usual D denotes the Kähler covariant derivative.

The calculation is a bit tedious, but completely straightforward and so we will present only the main steps in the following. First one can derive

$$D_i W = \frac{1}{4}\mathcal{K}_{ij} T^j + \frac{i}{2}\frac{\mathcal{K}_i}{\mathcal{V}} W \ . \tag{5.20}$$

Using now the formulae from section 4.2 one easily finds that

$$D_i W \overline{(D_j W)} g^{\bar{j}i} = \frac{1}{4}(-\tau \mathcal{V} \mathcal{K}_{ij} + \frac{2}{7}\tau^2 \mathcal{K}_i \mathcal{K}_j) T^i \bar{T}^j - \frac{i}{2}\tau \mathcal{K}_i (T^i \bar{W} - \bar{T}^i W) + 7|W|^2 \ . \tag{5.21}$$

Furthermore, one shows that

$$\frac{i}{2}\tau\mathcal{K}_i(T^i\bar{W} - \bar{T}^i W) = -7\tau\mathcal{V}Re\, W + 4(Im\, W)^2\ . \tag{5.22}$$

Finally, one obtains

$$4|W|^2 - \frac{i}{2}\tau\mathcal{K}_i(T^i\bar{W} - \bar{T}^i W) = \frac{1}{16}\left[(a^i a^j \mathcal{K}_{ij})^2 - (s^i s^j \mathcal{K}_{ij})^2\right]\ . \tag{5.23}$$

Putting all the results together one obtains the final form of the potential

$$V = \frac{1}{16}\left[-21\frac{\tau^2}{\mathcal{V}} + \frac{1}{\mathcal{V}^3}(a^i a^j \mathcal{K}_{ij})^2 + 16\frac{\tau^2}{\mathcal{V}}a^i a^j g_{ij}\right]\ , \tag{5.24}$$

which is precisely the potential derived in (5.14) from the compactification side.

To conclude, we have shown in this section that the compactification of M-theory on a manifolds with weak G_2 holonomy leads to an $N = 1$ supergravity coupled to chiral superfields in four dimensions with Kähler potential defined by (5.15) and superpotential (5.17). This is a nice test of the general analysis of M-theory compactifications on manifolds with G_2 structure presented in section 3 where the superpotential was derived from computing the four-dimensional gravitino mass term.

5.4 Inclusion of non-vanishing flux

In the previous section, we didn't consider the contributions to the superpotential arising from the internal flux G, since this term has already been covered for G_2 holonomy in [57] and we do not expect this analysis to be any different in our case. It is still, however, necessary to consider the forms in which it will be appropriate to expand the flux G. Firstly, note that following the discussion of the previous section, we have expanded the three-form leading to four-dimensional scalars as

$$T_3 = \varphi + ia_3 = T^i\Pi_i\ . \tag{5.25}$$

It is then clear that the part of the superpotential arising from G-flux in (3.22) takes the form

$$W_{\text{flux}} = \frac{1}{4}\int G\wedge T_3 = \frac{1}{4}\langle *G, T\rangle\ , \tag{5.26}$$

where $\langle , \rangle$ denotes the inner product for forms. Clearly, this quantity will vanish unless $*G$ has the same eigenvalue of $Q = *d$ as T does.[7] This suggests naïvely that we should expand G in the $*\Pi_i$, however these forms will not in general be independent of the choice of the metric. It should nevertheless be possible to find linear combinations of the $*\Pi_i$ that form a dual basis $\{\widetilde{\Pi}^i\}$ obeying

$$\int \Pi_i \wedge \widetilde{\Pi}^j = \delta_i^j\ . \tag{5.27}$$

We then expand $G = 4G_i\widetilde{\Pi}^i$. Note that turning on this flux will in general either break supersymmetry or introduce warping into the compactification, so it is usual to consider G as somehow 'small'.

[7] Note that the operator Q acting on 3-forms on a seven-dimensional manifold is self-adjoint as $Q^\dagger = (d*)^\dagger = *d^\dagger = **d* = d* = Q$.

This analysis produces a final form for the superpotential of

$$W = \Lambda + G_i T^i + k_{ij} T^i T^j \ , \tag{5.28}$$

for constant Λ, G_i, k_{ij}. The study of superpotentials of this form arising from compactification of M-theory on manifolds of G_2 structure is of phenomenological interest in its own right and is a question that we hope to return to in a later publication. In particular we would like to extend the analysis of [9] to see if vacua of small or vanishing cosmological constant can be obtained for the case of weak G_2 compactifications of M-theory.

6 Conclusions

In this paper we have analysed M-theory compactifications on manifolds with G_2 structure. Using the globally defined spinor which exists on such manifolds one can define the four-dimensional gravitino and compute explicitly the terms which give rise to the gravitino mass term in four dimensions. From this we were able to derive the general form for the superpotential which appears in M-theory compactifications on manifolds with G_2 structure (3.22). This formula generalises in a natural way the one which was derived for manifolds with G_2 holonomy in [7] which was derived based on the conjecture made in [52].

However, even if such a compact formula can be written for the superpotential, its expression in terms of the low energy fields is not known unless one specifies further the structure of the internal manifold. This was the purpose of the second part of this paper where we derived the effective action that appears from compactifications of M-theory on manifolds with weak G_2 holonomy. It turns out that the possible metric variations on such manifolds are in one to one correspondence with the three-forms on the weak G_2 manifold satisfying

$$d\alpha = -\tau * \alpha \ . \tag{6.1}$$

It also turns out that fluctuations of the three-form field $\hat{A}_3$ that are proportional to such forms lead to scalar fields in four dimensions that are massless in the background AdS solution. Thus, as for the case of manifolds with G_2 holonomy, the metric fluctuations compatible with the structure and the massless modes of the three-form field $\hat{A}_3$ pair up into complex fields which will be the scalar components of the chiral superfields in four dimensions. The superpotential appears to be quadratic in the superfields (5.17) and we have shown by an explicit calculation that the potential which can be derived from this superpotential matches the one which appears from the compactification on the weak G_2 manifold.

The phenomenological viability of the model we have constructed here is not very clear at the moment, mainly because of the large AdS curvature of the four-dimensional space. However, there are several directions which are worth investigating. The first of these would be a systematic study of the mass operators (5.4) in AdS backgrounds, which may also admit stable states of mass$^2 \leq 0$ that we have not considered here. Secondly the appearance of a quadratic superpotential opens up a new set of possibilities for finding a minimum of the potential as well as BPS states in the four-dimensional supergravity such as [39].

Furthermore, explicit examples of weak G_2 compactifications may be relevant in the process of obtaining the Standard Model spectrum from M-theory. It is well known that for chiral fermions to appear one needs conical singularities on the internal space [10, 11]. However, until now there was

no explicit construction of a *compact* manifold with G_2 holonomy which contains such singularities. On the other hand, the only explicit example of a compact manifold with G_2 structure which has such singularities is the case constructed in [18] and which is a weak G_2 manifold. Considering an explicit example would be interesting both in terms of looking for supersymmetric minima and BPS states in that model and also for studying anomaly cancellation.

In view of the work in [34, 38] it would be interesting to perform a similar analysis to that above in the case of weak $SU(3)$ holonomy, where we expect to find analogous results.

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Appendix

A Conventions and notation

Throughout the paper we use the following notation. Uppercase indices $M, N, \ldots = 0, \ldots, 10$ denote curved eleven-dimensional indices. Four-dimensional indices are denoted by Greek letters $\mu, \nu, \ldots = 0, \ldots, 3$ and we use lowercase letters $m, n, \ldots = 1, \ldots, 7$ for the indices on the internal manifold. Finally, indices $i, j \ldots$ are used to label the directions on the deformation space of manifolds with weak G_2 holonomy.

Where possible we use the index-free (form) notation. The relation the tensor notation is given by

$$A_{(p)} = \frac{1}{p!} A_{m_1 \ldots m_p} dx^{m_1} \wedge \ldots dx^{m_p} \ . \tag{A.1}$$

In our conventions, the standard operations on forms in D dimensions are given by

$$\begin{aligned} (dA)_{m_1 \ldots m_{p+1}} &= (p+1) \nabla_{[m_1} A_{m_2 \ldots m_{p+1}]} \\ (d^\dagger A)_{m_1 \ldots m_{p-1}} &= -\nabla^n A_{n m_1 \ldots m_{p-1}} \\ (*A)_{m_1 \ldots m_{D-p}} &= \frac{(-1)^{p(D-p)}}{p!} \epsilon_{m_1 \ldots m_{D-p}}{}^{n_1 \ldots n_p} A_{n_1 \ldots n_p} \end{aligned} \tag{A.2}$$

where we use the conventions that the ϵ-symbol is a proper tensor (rather than a tensor density) which is normalised as

$$\epsilon_{01 \ldots D-1} = +\sqrt{|\det(g)|} \ . \tag{A.3}$$

The symbol g usually denotes the metric as follows: g_{MN}, g_{mn} and $g_{\mu\nu}$ are space-time metrics while g_{ij} denotes the metric on the deformation space.

Finally we use the following conventions for the gamma-matrices. The eleven-dimensional gamma-matrices $\{\Gamma_M\}$ satisfy the Clifford algebra

$$\{\Gamma_M, \Gamma_N\} = 2 g_{MN} \ , \tag{A.4}$$

and are taken to be real. Furthermore, they are chosen to satisfy

$$\Gamma_0\Gamma_1 \ldots \Gamma_{10} = 1 \ , \tag{A.5}$$

or equivalently

$$\epsilon^{M_1 \ldots M_{11}} \Gamma_{M_1} \ldots \Gamma_{M_{11}} = -11! \tag{A.6}$$

According to (2.4) we decompose the gamma matrices as

$$\begin{aligned} \Gamma_\mu &= \gamma_\mu \otimes \mathbb{1} \ , \\ \Gamma_m &= \gamma \otimes \gamma_m \ . \end{aligned} \tag{A.7}$$

$\{\gamma_\mu\}$ are the four-dimensional gamma matrices which are also chosen to be real, while $\{\gamma_m\}$ are the gamma matrices on the internal manifold and are chosen purely imaginary. Note that this choice of reality of the gamma matrices is consistent with the fact that the the eleven-dimensional ones are real as the four-dimensional chirality matrix γ is defined as

$$\gamma = i\gamma_0 \ldots \gamma_3 \in i\mathbb{R} \ , \tag{A.8}$$

and is purely imaginary. Last, the gamma matrices γ_m on the internal manifold satisfy

$$\gamma_1 \ldots \gamma_7 = i \ . \tag{A.9}$$

Our conventions for spinor conjugation are that, for general spinor ψ, in Minkowskian-signature spaces we have

$$\overline{\psi} := \psi^\dagger \gamma^0 \ , \tag{A.10}$$

where † denotes Hermitian conjugation and in Euclidean-signature spaces

$$\overline{\psi} := \psi^\dagger \ . \tag{A.11}$$

B G_2 structures

In this appendix we review the most important features about G_2 structures which we need in the calculations in the paper.

B.1 General properties

We consider the manifolds with G_2 structure to be seven dimensional manifolds which admit a globally defined, nowhere-vanishing spinor which we denote by η. Without restricting the generality of the setup we consider this spinor to be Majorana and we normalise it as

$$\overline{\eta}\eta = 1 \ . \tag{B.1}$$

It is well-known that one can always find a connection—which in general will not be torsion-free—that makes the globally defined spinor covariantly constant:

$$D_m^{(T)}\eta = D_m\eta - \frac{1}{4}\kappa_{mnp}\gamma^{np}\eta = 0 \ , \tag{B.2}$$

where D_m denotes the covariant derivative with respect to the Levi-Civita connection and κ_{mnp} is the contorsion.

Using the spinor η one can construct a globally defined and nowhere vanishing totally antisymmetric tensor

$$\varphi_{mnp} = i\eta^T \gamma_{mnp}\eta \ , \tag{B.3}$$

where γ_{mnp} denotes the antisymmetric product of three gamma matrices with unit norm. The Hodge dual of the form φ can be written similarly

$$\Phi_{mnpq} = (*\varphi)_{mnpq} = \frac{i}{3!}\epsilon_{mnpq}{}^{rst}\eta^T\gamma_{rst}\eta = -\eta^T\gamma_{mnpq}\eta \ . \tag{B.4}$$

It is straightforward to check using (B.2) that both φ and Φ are covariantly constant with respect to the connection with torsion, i.e.

$$\begin{aligned} \nabla^{(T)}_m \varphi_{npq} &:= \nabla_m \varphi_{npq} - 3\kappa_{m[n}{}^r \varphi_{pq]r} = 0 \ , \\ \nabla^{(T)}_m \Phi_{npqr} &:= \nabla_m \Phi_{npqr} + 4\kappa_{m[n}{}^s \Phi_{pqr]s} = 0 \ , \end{aligned} \tag{B.5}$$

Note that these are the only forms which can be constructed from spinor bilinears as the combinations with antisymmetric products of one, two, five and six gamma matrices vanish identically. Using the φ one can write the volume of the manifold with G_2 structure to be

$$\mathcal{V} = \frac{1}{7}\int_{K_7} \varphi \wedge *\varphi = \frac{1}{7}\int_{K_7} \varphi \wedge \Phi \ . \tag{B.6}$$

In addition to the covariantly constant spinor, η, there are seven spinor directions left which we denote η_m. These obey

$$\begin{aligned} \overline{\eta}\eta_m = \overline{\eta}_m\eta &= 0 \ , \\ \overline{\eta}_m\eta^n &= \delta^n_m \ . \end{aligned} \tag{B.7}$$

The action of antisymmetrised products of gamma matrices on η is given by

$$\begin{aligned} \gamma^m\eta &= ig^{mn}\eta_n \\ \gamma^{mn}\eta &= \varphi^{mnp}\eta_p \\ \gamma^{mnp}\eta &= -i\varphi^{mnp}\eta + i\Phi^{mnpq}\eta_q \\ \gamma^{mnpq}\eta &= -\Phi^{mnpq}\eta - 4\varphi^{[mnp}g^{q]r}\eta_r \\ \gamma^{mnpqr}\eta &= -i\left(\Phi^{[mnpq}g^{r]s} + 4\varphi^{[mnp}\varphi^{qr]s}\right)\eta_s \\ \gamma^{mnpqrs}\eta &= -\left(\Phi^{[mnpq}\varphi^{rs]t} + 4\varphi^{[mnp}\Phi^{qrs]t}\right)\eta_t \\ \gamma^{mnpqrst}\eta &= i\epsilon^{mnpqrst}\eta - i\left(\Phi^{[mnpq}\Phi^{rst]u} + 4\varphi^{[mnp}\Phi^{qrs}{}_v\varphi^{t]uv}\right)\eta_u \ . \end{aligned} \tag{B.8}$$

By using the above, together with (B.7), Fierz identities and Dirac matrix identities such as those

in [62] one can derive some useful formulae relating the forms φ and Φ

$$\begin{aligned}
\varphi_{mnr}\varphi^{rpq} &= \Phi_{mn}{}^{pq} + 2\delta^{pq}_{mn} && \text{(B.9)}\\
\varphi^{mns}\Phi_{spqr} &= 6\delta^{[m}_{[p}\,\varphi^{n]}{}_{qr]} && \text{(B.10)}\\
\Phi_{mnpt}\Phi^{qrst} &= 6\delta^{mnp}_{qrs} + 9\Phi_{[mn}{}^{[qr}\,\delta^{s]}_{p]} - \varphi_{mnp}\varphi^{qrs} && \text{(B.11)}\\
36\delta^{[rs}_{[mn}\,\varphi_{pq]}{}^{t]} &= \Phi_{mnpq}\varphi^{rst} + 4\varphi_{[mnp}\,\Phi_{q]}{}^{rst} - \epsilon_{mnpq}{}^{rst} && \text{(B.12)}\\
24\varphi^{[tu}{}_{(m}\delta^{v]}_{n)} &= \epsilon^{pqrstuv}\varphi_{mpq}\varphi_{nrs} && \text{(B.13)}\\
\epsilon_{mnpqrst} &= 5\varphi_{[mnp}\Phi_{qrst]}\ , && \text{(B.14)}
\end{aligned}$$

where $\delta^{n_1\ldots n_a}_{m_1\ldots m_a} := g_{[m_1}{}^{[n_1}\cdots g_{m_a]}{}^{n_a]}$. Further identities may be derived by contracting indices in the above.

B.2 Classification of G_2 structures

As we said before, manifolds with G_2 structure are characterised by the existence of a globally defined spinor or equivalently by a three form φ. Also, as we shall see in the next section, the structure defines a metric via (B.20). However, in general, neither the spinor η nor the three-form φ are covariantly constant with respect to the Levi–Civita connection defined by this metric. The failure of this connection to preserve the structure defines the intrinsic torsion and measures the deviation of the G_2 structure from the G_2 holonomy case. It is well known [15] that in general the intrinsic torsion is a one-form with values in the orthogonal complement of the structure group in $SO(n)$. For the structure group G_2 this can be encoded in the following expression

$$T^0 \in \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3 \oplus \mathcal{W}_4\ , \tag{B.15}$$

where the description of the torsion classes $\mathcal{W}_1, \ldots, \mathcal{W}_4$ is given in the table 1. We have used the superscript 0 to denote degrees of freedom 'left-over' from other projections.

Class	Dimension	Interpretation
$\mathcal{W}_1$	**1**	$d\varphi \llcorner \Phi$
$\mathcal{W}_2$	**7**	$\varphi \llcorner d\varphi, \varphi \llcorner d * \varphi$
$\mathcal{W}_3$	**14**	$(d * \varphi)^0$
$\mathcal{W}_4$	**27**	$(d\varphi)^0$

Table 1: Description of the torsion classes of manifolds with G_2 structure.

Using the G_2 invariant form, φ, and its Hodge dual $\Phi = *\varphi$, one can define projection operators which select some particular representation of the group G_2. In particular we will be mostly interested into the projector on the singlet subspaces. For example the torsion component which is in $\mathcal{W}_1$ is defined as

$$(P_1 T)_{mnp} = T^1 \varphi_{mnp}\ , \tag{B.16}$$

where

$$T^1 = \frac{\int \sqrt{g}\, T_{mnp}\varphi^{mnp}}{\int \sqrt{g}\, \varphi_{mnp}\varphi^{mnp}} = \frac{1}{42\mathcal{V}} \int \sqrt{g}\, T_{mnp}\varphi^{mnp}\ . \tag{B.17}$$

Using (B.5) and (B.10) one can compute

$$\begin{aligned} d\varphi \wedge \varphi = d\varphi \wedge *\Phi &= \frac{\sqrt{g}}{6}\nabla_{[m}\varphi_{npq]}\Phi^{mnpq} = \frac{\sqrt{g}}{2}\kappa_{[mn}{}^{r}\varphi_{pq]r}\Phi^{mnpq} \\ &= 2\sqrt{g}\kappa_{mnp}\varphi^{mnp} = 2\sqrt{g}T_{mnp}\varphi^{mnp} \ , \end{aligned} \tag{B.18}$$

and so T^1 takes now the form

$$T^1 = \frac{1}{84\mathcal{V}}\int d\varphi \wedge \varphi \ . \tag{B.19}$$

B.3 Induced metric

As we pointed out before, a globally defined three-form on a seven-dimensional space assures that the manifold has G_2 structure. Moreover, given such a three-form φ, a metric is also uniquely defined by

$$\begin{aligned} g_{mn} &= (\det(s))^{-1/9}\, s_{mn} \ , \\ s_{mn} &= \frac{1}{144}\varphi_{mp_1p_2}\varphi_{np_3p_4}\varphi_{p_5p_6p_7}\hat{\epsilon}^{p_1\dots p_7} \ , \end{aligned} \tag{B.20}$$

where $\hat{\epsilon}^{m_1\dots m_7} = \sqrt{\det(g)}\epsilon^{m_1\dots m_7} = \pm 1$. Thus one can parameterise the deformations of the metric δg_{mn} in terms of the deformations of the G_2-structure $\delta\varphi$. Using the above definition of the metric and the relations (B.10) one immediately finds

$$\delta s_{mn} = \frac{\sqrt{\det(g)}}{2}\varphi_{(m}{}^{pq}\delta\varphi_{n)pq} \ . \tag{B.21}$$

With this the metric variation δg_{mn} becomes

$$\delta g_{mn} = \frac{1}{2}\varphi_{(m}{}^{pq}\delta\varphi_{n)pq} - \frac{1}{18}(\varphi\llcorner\delta\varphi)g_{mn} \ . \tag{B.22}$$

Here, as elsewhere in this paper, $\llcorner$ denotes the contraction of indices, so that for p-form α and q-form β obeying $q < p$,

$$(\beta\llcorner\alpha)_{m_1\dots m_{q-p}} := \beta_{n_1\dots n_q}\alpha^{n_1\dots n_q}{}_{m_1\dots m_{q-p}} \ . \tag{B.23}$$

B.4 Metric on the deformation space

Let us suppose that we have expanded φ in terms of some parameters s^i

$$\varphi = s^i\Pi_i \ , \tag{B.24}$$

where Π_i are three forms which further satisfy

$$P_7\Pi_i = 0 \tag{B.25}$$

and which are supposed not to depend on the parameters s^i. The meaning of the above equation is that using the set of forms Π_i one can parameterise the deformations of the G_2-structure φ in terms of variations of the parameters s^i. These parameters, or more correctly their variations, are constant on the internal manifold, but from a four-dimensional perspective they become (scalar) fields. We have seen in the previous section that variations of the G_2 structure induce variations

in the metric on the manifold via (B.22) and so we can say that the parameters s^i describe the metric fluctuations on the internal manifold. Consequently the kinetic term for these fields in four dimensions appears from the expansion of the eleven-dimensional Ricci scalar. It is well-known that this leads to the following term

$$\int_{M_{11}} \sqrt{-g}R_{11} = \int_{M_{11}} \sqrt{-g}\mathcal{V}\left\{R_4 + R_7 + \frac{1}{4\mathcal{V}}\left[\partial_\mu g_{mn}\partial^\mu g^{mn} - \mathrm{tr}(\partial_\mu g)\mathrm{tr}(\partial^\mu g)\right]\right\} . \tag{B.26}$$

Inserting (B.22) in the above equation and using (B.11) one obtains

$$\begin{aligned} S_{\mathrm{kin}} &= -\frac{1}{2}\int_{M_4} \sqrt{-g}\partial_\mu s^i \partial^\mu s^j \int_{K_7} \sqrt{g}\frac{1}{4}\left[\left(\frac{1}{2}\varphi_{(m}{}^{pq}\delta\varphi_{n)pq} - \frac{1}{18}(\varphi\llcorner\delta\varphi)g_{mn}\right)^2 - \frac{1}{81}(\varphi\llcorner\Pi_i)(\varphi\llcorner\Pi_j)\right] \\ &= -\frac{1}{4}\int_{K_7}\Pi_i \wedge *\Pi_j \int_{M_4}\sqrt{-g}\;\partial_\mu s^i\partial^\mu s^j + \frac{3}{2}\int_{M4}\sqrt{-g}\;\frac{\partial_\mu\mathcal{V}\partial^\mu\mathcal{V}}{\mathcal{V}^2} . \end{aligned} \tag{B.27}$$

To read off the sigma-model metric for the scalars s^i in four dimensions one has to take into account the redefinitions of the space-time metric by a volume factor (2.8). Note that under this rescaling the last term in the above equation disappears and we are left with

$$g_{ij} = \frac{1}{4\mathcal{V}}\int_{K_7}\Pi_i \wedge *\Pi_j . \tag{B.28}$$

Since the resulting four-dimensional action is supposed to be supersymmetric, the above metric has to be Kähler. This is indeed the case and the Kähler potential was derived on general grounds in section 3.2

$$K = -3\ln(\mathcal{V}) , \tag{B.29}$$

where the volume $\mathcal{V}$ was defined in (B.6). To show that this is the Kähler potential corresponding to the metric (B.28) we have to know the dependence of the the volume on the parameters s^i. φ depends linearly on s^i, (B.24) provided the forms Π_i are independent of these parameters. For the form Φ this dependence is more complicated because of the Hodge duality operation which is involved, but its variation with the parameters s^i can be read off from (4.1). One finds

$$\frac{\delta\mathcal{V}}{\delta s^i} = \frac{1}{7}\int_{K_7}\Pi_i\wedge\Phi + \frac{1}{7}\int_{K_7}\varphi\wedge *\frac{4}{3}\Pi_i = \frac{1}{3}\int_{K_7}\Pi_i\wedge\Phi . \tag{B.30}$$

With this, one immediately finds the first derivative[8] of the Kähler potential (B.29)

$$\begin{aligned} K_i := \partial_i K &= \frac{1}{2}\frac{\partial}{\partial s^i}\left(-3\ln(\mathcal{V})\right) \\ &= \frac{-1}{2\mathcal{V}}\int\Pi_i\wedge\Phi . \end{aligned}$$

Using again the relation (4.1) we can compute the second derivative of the Kähler potential

$$\begin{aligned} K_{i\bar{j}} &= \frac{1}{4}\left(\frac{\partial\mathcal{V}}{\partial s^j}\mathcal{V}^{-2}\int\Pi_i\wedge\Phi - \mathcal{V}^{-1}\int\Pi_i\wedge\left(\frac{4}{3}*P_1\Pi_j - *P_{27}\Pi_j\right)\right) \\ &= \frac{1}{4\mathcal{V}}\int\Pi_i\wedge *\Pi_j , \end{aligned} \tag{B.31}$$

[8]Note that a Kähler potential makes sense only in the context of complex geometry. Thus what we have in mind here is that the Kähler potential (B.29) is a function of the complex fields T^i defined in (5.18) and thus derivatives are then taken with respect to the fields T^i rather then only their imaginary parts s^i.

where we have used (B.25) and the last equality used (B.30). As anticipated one notices now that the metric (B.28) can be indeed derived from the Kähler potential (B.29). We should stress here that this result is quite general and holds as long as the forms Π_i do not depend on the parameters s^i and they satisfy (B.25).

C Weak G_2 holonomy

Usually not much can be said about generic manifolds with G_2 structure. The characterisation of such manifolds needs in general the introduction of other forms besides φ and Φ in order to parameterise the torsion (see for example [41]). However there is a simple case where such manipulations are not necessary. This is the case of manifolds with weak G_2 holonomy which are characterised by the fact that the intrinsic torsion resides completely within the first torsion class (see table 1), or in other words, it is a singlet under the G_2 structure group. We can thus write the contorsion

$$\kappa_{mnp} = \kappa \varphi_{mnp} \, , \tag{C.1}$$

where κ is a constant on the manifold with G_2 structure. Note that this is the only possibility we have as φ_{mnp} is the unique three-index object which is a singlet under G_2. It is clear that the contorsion is totally antisymmetric and thus it also coincides with the torsion tensor $T_{mnp} = \kappa_{[mn]p}$. Using (C.1) and the fact that φ is covariantly constant with respect to the connection with torsion we can compute the exterior derivative of φ

$$(d\varphi)_{mnpq} = 4\nabla_{[m}\varphi_{npq]} = 12\kappa_{[mn}{}^{r}\varphi_{pq]r} = 12\kappa\Phi_{mnpq} \, . \tag{C.2}$$

We thus see, as anticipated in the table 1, that the exterior derivative of φ is indeed again a singlet, namely Φ. It will be more convenient to introduce another parameter $\tau = 12\kappa$ such that

$$\kappa_{mnp} = \frac{\tau}{12}\varphi_{mnp} \, . \tag{C.3}$$

in terms of which the weak G_2 condition takes the more custom form

$$\begin{aligned} d\varphi &= \tau * \varphi \, , \\ d * \varphi &= 0 \, . \end{aligned} \tag{C.4}$$

Having defined the general properties of manifolds with G_2 structure we can now analyse the main features of the special case of manifolds with weak G_2 holonomy in the following.

C.1 Weak G_2 identities

Starting from (B.5) and using the relations (B.10) one can show that

$$\nabla_m\varphi_{npq} = 3\kappa_{m[n}{}^{r}\varphi_{pq]r} = \frac{\tau}{4}\Phi_{mnpq} \, . \tag{C.5}$$

Repeating the procedure for Φ one finds

$$\nabla_m\Phi_{npqr} = -4\kappa_{m[n}{}^{s}\Phi_{pqr]s} = -\frac{\tau}{3}\phi_{m[n}{}^{s}\Phi_{pqr]s} = -\tau g_{m[n}\varphi_{pqr]} \, . \tag{C.6}$$

From the above relation one immediately derives

$$\Box\varphi_{mnp} = \nabla_q \nabla^q \varphi_{mnp} = -\frac{\tau^2}{4}\varphi_{mnp} \ . \tag{C.7}$$

In the main text we also needed a couple of relations involving the curvature tensors of weak G_2 manifolds. To compute these we start from the fact that the globally defined spinor is covariantly constant with respect to the connection with torsion (B.2). Since we work with imaginary gamma matrices the spinor $\gamma^{pq}\eta$ is still a Majorana spinor and thus can be expanded in terms of a basis for Majorana spinors on the weak G_2 manifold $\{\eta, \eta_m\}$ as defined in §B.1. It is then straightforward to derive that

$$D_m\eta = \frac{1}{4}\kappa_{mnp}\gamma^{np}\eta = \frac{\tau}{8}\eta_m \ . \tag{C.8}$$

Taking the commutator of two covariant derivatives acting on the spinor η one obtains

$$R_{mnpq}\gamma^{pq}\eta = \frac{\tau^2}{8}\gamma_{mn}\eta \ . \tag{C.9}$$

Multiplying this from the left with γ^n and using the first Bianchi identity for the Riemann tensor immediately gives the Ricci curvature of weak G_2 manifolds as

$$R_{mn} \equiv R^p{}_{mpn} = \frac{3}{8}\tau^2 g_{mn} \ , \tag{C.10}$$

which shows that these manifolds are Einstein. Note, as a matter of fact, that the scaling behavior of τ found in (4.8) is in agreement with the above relation.

Multiplying again (C.9) by γ^s and contracting with η^T one obtains

$$R_{mnpq}\varphi^{pqs} = \frac{\tau^2}{8}\varphi_{mn}{}^s \ , \tag{C.11}$$

which can be used to prove other identities about the Riemann tensor in weak G_2.

C.2 More about weak G_2 manifolds

There are some more properties of weak G_2 which are rather important for the analysis presented in the paper and which we still have to prove. We will mainly be interested in three-forms which seem to play a key role in the manifolds we were discussing. Before we start there is a useful remark we should make about three-forms on seven-dimensional manifolds (see for example [1]): The eigenfunctions of the Laplace operator corresponding to a non-zero eigenvalue μ^2 are in one to one correspondence with the eigenfunctions of the operator $Q = *d$ corresponding to the eigenvalues $\pm\mu$. As the reader has probably already noticed the operator Q plays an important role in our analysis as the forms which are relevant for the deformations of weak G_2 manifolds are eigenfunctions of this operator corresponding to the eigenvalue $-\tau$. It is these forms we are going to analyse in the following. As mentioned in the main text, the forms we are interested in also do not contain a part which transforms under **7** under G_2. It would be interesting if one could prove the existence of such forms, but this goes beyond the scope of this paper and so for our purposes we will just assume that the forms with these desired properties do indeed exist.

Let us consider the forms Π_i as in the main text which satisfy

$$\begin{aligned} &\Delta\Pi_i = \tau^2\Pi_i \ , \\ &P_7\Pi_i = 0 \ \Leftrightarrow \ (\Pi_i)_{mnp}\Phi^{mnp}{}_r = 0 \ , \end{aligned} \tag{C.12}$$

and prove as we have stated that for such forms the projectors P_1 and P_{27} commute with the differential operator $Q = *d$. For this, we consider the quantity

$$(\Pi_i)_{mnp}\varphi^{mnp} \, , \tag{C.13}$$

and show that it does not depend on the coordinates of the internal manifold. To see this we compute

$$\nabla_m(\varphi_{npq}(\Pi_i)^{npq}) = \frac{\tau}{4}\Phi_{mnpq}(\Pi_i)^{npq} + \varphi_{npq}(d\Pi^i)_m{}^{npq} + 3\varphi_{npq}\nabla^n(\Pi_i)_m{}^{pq} \, . \tag{C.14}$$

Using (C.12), the first term clearly vanishes. Furthermore, we can choose without loss of generality that the forms Π_i are eigenfunctions of the $Q = *d$ operator with eigenvalue $\pm\tau$. With this the second term becomes proportional to $\Phi_{mnpq}(\Pi_i)^{npq}$ which again vanishes for the forms we consider. Thus, we are left with

$$\nabla_m(\varphi_{npq}(\Pi_i)^{npq}) = 3\varphi_{npq}\nabla^n(\Pi_i)_m{}^{pq} \, . \tag{C.15}$$

The right hand side can be computed if we push φ through the derivative as $d{*}\varphi = 0$ and then notice that the combination $\varphi_{npq}(\Pi_i)_m{}^{pq}$ is symmetric in the indices m, n which is again a consequence of (C.12). We obtain

$$\nabla_m(\varphi_{npq}(\Pi_i)^{npq}) = 3\nabla^n(\phi_{npq}(\Pi_i)_m{}^{pq}) = 3\nabla^n(\phi_{mpq}(\Pi_i)_n{}^{pq}) = 3\frac{\tau}{4}\Phi^n{}_{mpq}(\Pi_i)_n{}^{pq} \, , \tag{C.16}$$

which again vanishes upon using (C.12). Note that in the last relation we have also used that $d * \Pi_i = 0$ which holds true if we take the forms Π to be eigenfunctions of the operator Q. This completes the proof of

$$\nabla_m(\varphi_{npq}(\Pi_i)^{npq}) = 0 \, . \tag{C.17}$$

Since $P_1\Pi_i$ is defined as

$$(P_1\Pi_i)_{mnp} = \frac{1}{42}(\varphi_{qrs}(\Pi_i)^{qrs})\varphi_{mnp} \, , \tag{C.18}$$

we conclude that

$$[P_1, Q]\Pi_i = 0 \, . \tag{C.19}$$

Since for these forms it also holds that $P_1\Pi_i + P_{27}\Pi_i = \Pi_i$ it follows that

$$[P_{27}, Q]\Pi_i = 0 \, . \tag{C.20}$$

There is one more aspect which is crucial in the whole construction which we did in this paper, namely the dependence on the parameters s^i introduced in (4.13) of the basis of forms we consider Π_i. In principle there is no reason to believe that the solutions of the equation $\Delta = \tau^2$ are independent of the parameters s^i of the manifold as the metric itself depends on them. In fact when one does a variation of the metric the operator Δ changes and so we expect its solution to change as well. This also happens in ordinary manifolds with restricted holonomy like Calabi–Yau manifolds or manifolds with G_2 structure. In these cases however, one can easily show that such a dependence on the moduli of the harmonic forms is exact. If one assumes that the same happens for the case of forms which are eigenvalues of the Laplace operator corresponding to some non-zero eigenvalue then it is quite easy to show that such a 'mild' dependence on the parameters is not going to affect the results we have derived so far. First of all it is straightforward to see that this dependence drops out completely from the definition of $\mathcal{K}_{ij}$. The other thing to show is that the metric on the deformation space does not get an additional dependence on the parameters from

the forms Π_i. Indeed, if such a dependence on the parameters s^i of these forms is only via an exact form one can immediately see that

$$\int \delta\Pi_i \wedge *\Pi_j = \int d\beta_i \wedge *\Pi_j = -\int \beta_i \wedge d * \Pi_j = 0\ , \tag{C.21}$$

because the forms Π_i are coclosed. We thus conclude that the only relevant dependence on the parameters of the weak G_2 manifold s^i, is via the expansion (4.13) as we considered in the main text.

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